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A METHOD FOR COMPARING HEAT SINKS BASED ON REYNOLDS ANALOGY

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ABSTRACT

An alternative formulation of Reynolds analogy has been used to study the interaction between friction and convection in heat sinks. The formulation correlates the mechanical power needed to overcome the pressure losses with the heat dissipation. It can be applied to both external and internal flow. The analogy is accurate for flat or moderately curved surfaces. For more complex objects there are several disturbing impacts. An "efficiency" factor, called the analogy number, Λ, is introduced to compensate for these discrepancies. Approximate values for the objects covered are: plates ≈1.27, cylinders <0.8, parallel plates 0.5 – 0.9 and heat sinks <0.5.

The theory reveals a strong dependence on velocity. To minimise the mechanical power losses it is therefore important to keep the velocity as low as possible. This tendency is confirmed by comparisons with experimental data for heat sinks.

The fin shape issue is addressed. A proof that no fin shape offers any heat dissipation or weight advantage over straight fins is presented. For pressure losses it is concluded that straight fins perform better than any other fin shape if the in- and outlet losses are negligible. If not, it is likely that straight fins with slightly rounded corners at the inlet and a slightly decreasing fin thickness towards the outlet are advantageous.

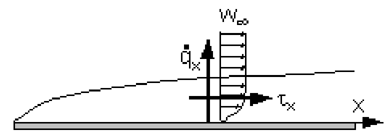


Figure 1. Local heat flux and shear stress.

1. REFORMULATION OF REYNOLDS ANALOGY

Fundamental heat transfer theory predicts a correlation between friction and convection. It is called Reynolds analogy and can be derived form boundary layer theory [4]. None of the conventional formulations are convenient for the purpose of this document. An alternative has therefore been developed. The fully correct derivation is somewhat complicated. A simplified approach is therefore used here. It is based on generally accepted equations for the local heat transfer coefficient and shear stress on flat isothermal plates, figure 1, [1]. The end result is the same for both turbulent and laminar flow. The latter equations

Nomenclature

A	surface [m^2]
c_p	specific heat [J/kgK]
C_d	drag factor [--]
D_e	hydraulic diameter [m]
$\dot{e}$	mechanical power per surface unit [W/m^2]
$\dot{E}$	mechanical power [W]
f	friction factor [--]
h	heat transfer coefficient [W/m^2K]
k	thermal conductivity [W/mK]
L	length [m]
L_c	characteristic length [m]
w	velocity [m/s]
$\dot{q}$	heat flux [[W/m^2]
$\dot{Q}$	heat flow [W]
Λ	analogy number [--]
Δp	pressure drop [N/m^2]
ρ	density [kg/ m^3]
τ	shear stress [N/m^2]
ν	dynamic viscosity [m^2/s]
ϑ	temperature difference [K]
η_t	thermal efficiency [--]
Ec	$w^2/(c_p \cdot \vartheta)$
Nu	$h \cdot L_c/k$
Re	$w \cdot L_c/\nu$
Pr	$\rho \cdot \nu \cdot c_p/k$
Gz	$Re \cdot Pr \cdot D_e/L$

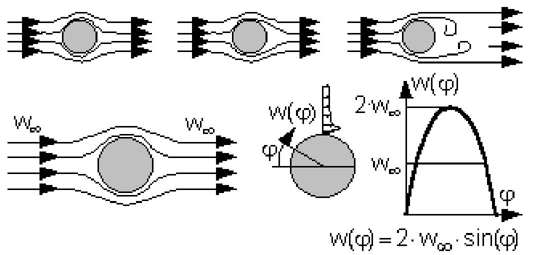


Figure 2. Velocity distribution around a cylinder as predicted by potential flow theory.

are used here

$$Nu_x = 0.332 \cdot Re_x^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}} \tag{1}$$

$$\tau_x = 0.332 \cdot \rho \cdot w_\infty^2 \cdot Re_x^{-\frac{1}{2}} \tag{2}$$

The heat flux can be derived form equation 1 by combining it with the definition of Nu and Newton's cooling equation

$$\dot{q}_x = 0.332 \cdot \frac{k \cdot \vartheta}{L} \cdot Re_x^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}} \tag{3}$$

The mechanical power requirement per surface unit equals the shear stress times the velocity

$$\dot{e}_x = 0.332 \cdot \rho \cdot w_\infty^3 \cdot Re_x^{-\frac{1}{2}} \tag{4}$$

Using the definitions for the Re and Pr and combining equations 3 and 4 results in

$$\frac{\dot{e}_x}{\dot{q}_x} = \frac{w_\infty^2}{c_p \cdot \vartheta} \cdot Pr^{\frac{2}{3}} \tag{5}$$

This is a local formulation of Reynolds analogy. A more elaborated approach would have shown that it also is valid for moderately curved surfaces, [4]. The definition of the Eckert number results in the alternative equation

$$\frac{\dot{e}_x}{\dot{q}_x} = Ec \cdot Pr^{\frac{2}{3}} \tag{6}$$

Given that the surface is a flat plate, all the terms on the right side of equation 5 are constants. A simple integration in the flow direction yields

$$\frac{\dot{E}}{\dot{Q}} = 1.0 \cdot \frac{w_\infty^2}{c_p \cdot \vartheta} \cdot Pr^{\frac{2}{3}} \tag{7}$$

The integration process is more complicated for cases where the temperature difference and the velocity vary. In many cases it is not possible at all. When it is, the outcome is an equation with the same basic structure as

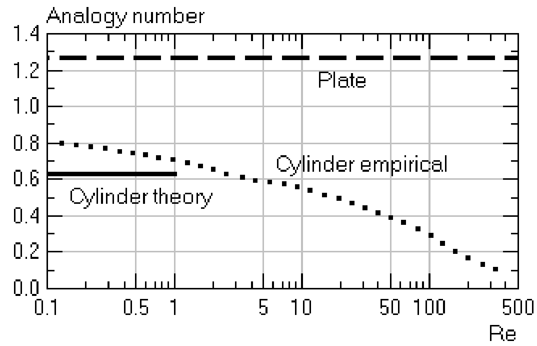


Figure 3. Comparison of Λ for plates and cylinders.

equation 7 but with another constant or an expression replacing that constant. An additional concern is that some mechanical power losses not are caused by friction and therefore result in no or little convection contribution. It is therefore convenient to define an efficiency factor called the analogy number, compare [2]

$$\Lambda = \frac{\dot{Q}}{\dot{E}} \cdot \frac{w_{ref}^2}{c_p \cdot \vartheta_{ref}} \tag{8}$$

When rearranged

$$\dot{E} = \frac{1}{\Lambda} \cdot \dot{Q} \cdot \frac{w_{ref}^2}{c_p \cdot \vartheta_{ref}} \tag{9}$$

This is a global formulation of Reynolds analogy. The velocity and the temperature difference need to be defined for each specific case. They have therefore been indexed "ref". By comparison with equation 7, the Λ-number for plates is found to be $Pr^{-2/3} \approx 1.27$.

One advantage with this formulation is that it can be used for both internal and external flow. Another advantage is that it explicitly references important design parameters. It is in particular the strong velocity dependence that is of interest for the following discussion.

2. FLOW AROUND CYLINDERS

The flow pattern around a cylinder is highly dependent on velocity, or more precisely Reynolds number, figure 2. There is in particular considerable vertex formation for high Re conditions. These vertexes consume mechanical power but contribute little to convection. They consequently tend to decrease the Λ-number. A theoretical approach that only accounts for friction can therefore only be approximately correct for low Re conditions. Assuming that the tangential velocity is given by potential flow theory, [3] figure 2, that the heat transfer coefficient distribution is uniform and integrating equation 5 results in $\Lambda = 0.5 \cdot Pr^{-2/3} \approx 0.63$. It is half of that for plates. The reason is that the tangential velocity varies considerably

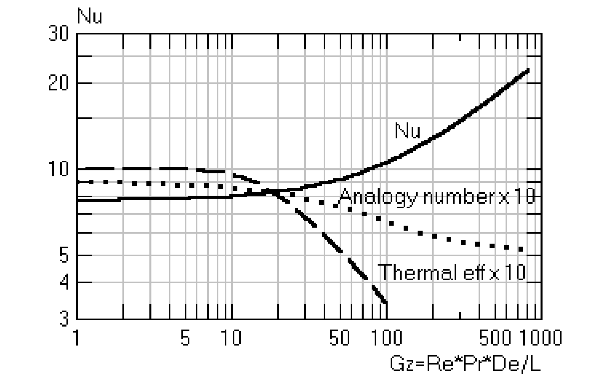


Figure 4. Nu, Λ and η_t for isothermal parallel plates.

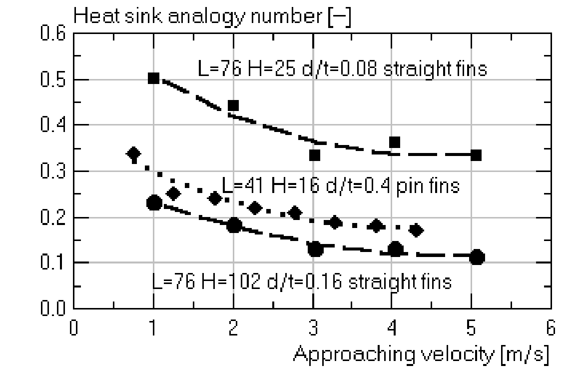


Figure 5. Comparison of heat sinks. d/t denotes fin thickness to pitch ratio.

and has two times the approaching velocity as its maximum value. Considering that the reference velocity is the approaching velocity and that the analogy is strongly velocity dependent, it is natural that the Λ-number for a cylinder is lower than for a plate.

A more applied approach to the problem is to extract the Λ-number from empirical data for convection and drag. It is in this case convenient to give the analogy a more conventional formulation based on generally accepted dimensionless representations. A derivation based on equation 8 results in

$$\Lambda = \frac{2 \cdot \pi}{C_d} \cdot \frac{Nu}{Re \cdot Pr} \qquad (10)$$

Values for C_d and Nu can for example be found in [1]. Figure 3 shows the result. The simplified theoretical approach is by no means exact but it does indicate the approximate level of the Λ-number for low Re conditions.

3. FLOW BETWEEN PARALLEL PLATES

The Λ-number for flow between parallel plates is lower than that for single plates. There are two reasons. The first is that the reference velocity is the average velocity in the gap and not the approaching velocity. The second is that mechanical power not only is needed to overcome friction but also to develop the velocity profile. Theories that cover this case are by no means elementary. An empirical approach is therefore used here. As above it can be derived from equation 8

$$\Lambda = \frac{8}{f} \cdot \frac{Nu}{Re \cdot Pr} \qquad (11)$$

The constant can vary depending on the definition of the friction factor and the characteristic length. Figure 4 shows Nu, Λ and η_t as functions of Gz for laminar flow and isothermal conditions. The curves are based on correlations for convection and friction found in [5]. More recent data but for a smaller validity range can be found in [6] [7]. Λ is a unique function of Gz for all correlations referenced. The not fully developed region is to the right in the diagram. It is characterised by high Nu-numbers and low Λ-numbers. The tendency for the fully developed region is the reverse. The thermal efficiency is defined as in figure 7. It is also a unique function of Gz

$$\eta_t = 1 - e^{-\frac{4 \cdot Nu}{Gz}} \qquad (12)$$

Typical values for heat sinks are $0.5 < \eta_t < 0.9$ which corresponds to $0.7 < \Lambda < 0.9$.

4. HEAT SINKS

The definition of the Λ-number is general and can be used for all heat transfer surfaces provided that the velocity and the temperature difference are given unambiguous definitions. For heat sinks it is convenient to use the approaching velocity and the mean logarithmic temperature difference. The approaching velocity can be defended as the ratio of the airflow and the cross section of the heat sink

$$w_{ref} = \frac{\dot{V}}{A_f} \qquad (13)$$

A_f is typically the product of the fin height and the heat sink width. For some cases it can nevertheless be purposeful to include an additional fin spacing on each side of the heat sink. The mechanical power needed to push the air through the heat sink is given by

$$\dot{E} = \Delta p \cdot \dot{V} \qquad (14)$$

Combining equations 13 and 14 with the definition of Λ, equation 8, results in

$$\Lambda_{hs} = \frac{\dot{Q} \cdot \dot{V}}{\Delta p \cdot A_f^2 \cdot c_p \cdot \vartheta_m} \qquad (15)$$

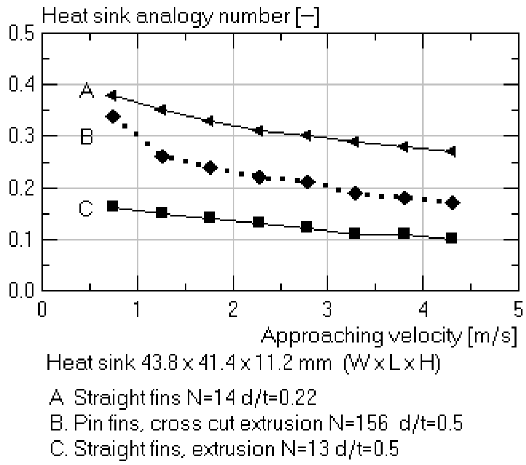
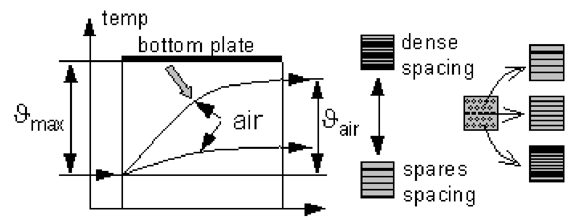


Figure 6. Comparison of heat sinks with the same outer measures.



$$\eta_t = \frac{\vartheta_{air}}{\vartheta_{max}}$$

Figure 7. Definition of thermal efficiency.

This equation can be applied to any heat sink regardless of fin shape. Figure 5 shows some examples from the Powerfin and the Heat Technology companies. A general tendency is that the Λ-number decreases with velocity. This effect is partly caused by a decline of the fin efficiency and partly by a decline of the Λ-number for parallel plates as Gz increases, figure 4. Another effect is a dependence of the fin thickness to pitch ratio, (d/t). It reflects that high d/t ratios result in high internal velocities.

The concept is particularly useful for comparisons of heat sinks with the same outer measures, figure 6. All three heat sinks have in this case roughly the same thermal resistance. Case B is based on measured data. Case A and C are calculated with the Hsink code, which has an uncertainty on the 20% level. This imperfection can therefore not change the general tendencies. Heat sink C is a conventional extruded design with straight uninterrupted fins. Heat sink B is a cross cut version of heat sink C. The air void is larger and the internal velocities are lower, which results in a significant improvement. Heat sink A has straight uninterrupted fins and the same total fin volume as heat sink B. The internal velocities are further lowered and the Λ-number is increased.

This example shows that the Λ-number favourably can be used to compare heat sinks. It should however be observed that the results like this not can be used for general fin shape comparisons.

5. FIN SHAPE COMPARISONS

The Λ-number concept is only one of the elements needed for a more profound discussion about fin shapes. In addition there must be a clear idea of which fin spacing, fin thickness etc, that result in conclusive comparisons. This issue can be clarified by an airside approach, figure 7. The thermal efficiency is defined as the ratio of the air temperature increase and the available temperature difference. It is evidently confined to the interval 0% - 100%.

If it is assumed that a fin material can have infinite conductivity it would be possible to create a heat sink with a large number of negligibly thin straight fins. The result would in that case be a heat sink with near 100% thermal efficiency. The fin material can alternatively be placed in one single fin and the result would in that case be a heat sink with near 0% thermal efficiency. Given a pin fin heat sink of the same material, it must therefore always be possible to reshape the fins as straight fins and find a particular material distribution that results in exactly the same thermal efficiency, figure 7. If the material has finite conductivity this conclusion does not change because both cases have the same total fin cross section and therefore also the same conduction losses.

It is evidently possible to reverse the proof and show that pin fins can be made as effective as straight fins. The procedure can in fact be used for any other fin shape. The essence of the proof is therefore that no fin shape has any advantage over any other fin shape regarding the two qualities heat dissipation and weight. There are of coarse validity restrains. The fins must have the same shape bottom to top and the approaching airflow must be confined to the heat sink.

Another important quality is friction losses. The fin shape does have an impact on this property. Figure 8 shows a comparison of three heat sinks. They all have the same thermal resistance and the same fin volume. They therefore also have the same air void. If the wavy flow channels in the middle heat sink are stretch out and placed side by side they form a rectangle. The channel velocity is inversely proportional to the width of that rectangle and therefore proportional to the channel length. Considering the strong velocity dependence in equation 9 it is obvious that straight fins have lower friction losses than wavy fins.

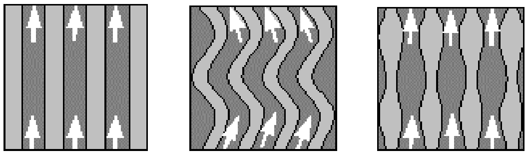


Figure 8. Low and uniform air velocities induce the lowest friction losses.

The right heat sink in figure 8 has approximately the same average channel velocity as the straight fin case but there are large velocity variations. Sine the analogy operates on the square of the velocity it is apparent that the low friction losses in the low velocity regions not can compensate for the high friction losses in the high velocity regions. The total friction losses must therefore be higher than for the straight fin case.

Other fin arrangements can be regarded as combinations of the cases discussed. None of them can therefore perform better than straight fins. Empirical data supporting this conclusion can be found in [8].

6. SECONDARY FIN SHAPE IMPACTS

There are a couple of secondary impacts that to some extent can modify the conclusion above. In- and outlet pressure losses is one of them. They are typically <15% of the total loss but they can be considerably higher for extreme cases and particularly for staggered fin arrays.

Rounding the corners at the fin fronts can reduce the inlet losses. Successively reducing the fin thickness towards the outlet can reduce the outlet losses, figure 9. These measures will however create deviations from the ideal shape for friction. It is therefore reasonably only a fraction of the in- and outlet losses that in this way can be transferred into an increased Λ-number. The gains should be on the percentage level for typical cases but they could be significant for extreme cases.

Staggered fin arrangements are exceptional because there are many in- and outlet losses. The gain of modifying the fin shape might therefore motivate the effort. A possible outcome of an optimisation process is the shape shown in figure 9. An interesting observation is that this shape resembles the elliptical shape used by some manufacturers.

It should nevertheless be underlined that the Λ-number for staggered arrays always is lower than for comparable uninterrupted straight fins. There are several reasons. The in- and outlet losses are higher, the flow path has a wavy pattern and the fins downstream row one are approached by high middle channel air velocities.

Another impact of secondary order is that the fin efficiency can be improved by making the fins slightly thicker at the inlet. This effect is caused by the fact that the heat transfer coefficient is higher at the inlet than at the outlet. A coarse estimate reveals that the possible gains for typical designs must be on the percentage level.

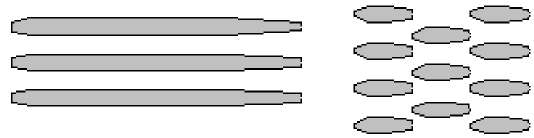


Figure 9. Fin shapes that reduce in- and outlet losses.

7. DISCUSSION

The theory presented does not cover non-confined flow. One of the problems with this type of flow is that the entering air can originate from many different directions. The only case that reasonably resembles confined flow is face on flow. The difficulty with this particular case is that some air leaves the heat sink from the top or the sides. Equation 15, which is based on the mean logarithmic temperature difference, can therefore not be used. An important issue in this context is if fin arrangements that allow air to escape from the sides of a heat sink can perform better than straight uninterrupted fins. Empirical data does not seem to be decisive on this point [9].

A limitation with the fin shape discussion is that it only covers fins that have the same shape bottom to top. It is however well known that tapered or trapezoidal fin profiles are advantageous, particularly if the fin efficiency is low [4]. This case is evidently much more complex than the case addressed in this document. More research is needed to clarify the subject.

Applied heat sink optimization is a complicated procedure that involves more considerations than covered by this document. The cost issue is always central but other matters, for example sensitivity to dust deposits, can some times also be important. In any applied context it is nevertheless always interesting to appraise the degree of physical performance compromise that has to be made. The fact that straight fins, which favorably can be calculated with relatively simple and fast analytical procedures, always offer the best performance simplifies this issue considerably.

8. REFERENCES

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